Existence and improved regularity for a nonlinear system with collapsing ellipticity

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Abstract. We study a nonlinear system made up of an elliptic equation of blended singular/degenerate type and Poisson's equation with a lowly integrable source. We prove the existence of a weak solution in any space dimension and, chiefly, derive an improved $\mathcal{C}^{1,\text{Log-Lip}}$ regularity estimate using tangential analysis methods. The system illustrates a sophisticated version of the proverbial thermistor problem and our results are new even in simpler modelling scenarios.

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