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Sharp Poincaré inequalities under Measure Contraction Property

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Abstract. We prove a sharp Poincaré inequality for subsets Ω of (essentially non-branching) metric measure spaces satisfying the Measure Contraction Property MCP(K, N), whose diameter is bounded above by D. This is achieved by identifying the corresponding one-dimensional model densities and a localization argument, ensuring that the Poincaré constant we obtain is best possible as a function of K, N and D. Another new feature of our work is that we do not need to assume that Ω is geodesically convex, by employing the geodesic hull of Ω on the energy side of the Poincaré inequality. In particular, our results apply to geodesic balls in ideal sub-Riemannian manifolds, such as the Heisenberg group.

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