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Eigenvalue asymptotics for weighted Laplace equations on rough Riemannian manifolds with boundary

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This work is dedicated to the memory of Alan McIntosh

Abstract. Our topological setting is a smooth compact manifold of dimension two or higher with smooth boundary. Although this underlying topological structure is smooth, the Riemannian metric tensor is only assumed to be bounded and measurable. This is known as a *rough Riemannian manifold.* For a large class of boundary conditions we demonstrate a Weyl law for the asymptotics of the eigenvalues of the Laplacian associated to a rough metric. Moreover, we obtain eigenvalue asymptotics for weighted Laplace equations associated to a rough metric. Of particular novelty is that the weight function is not assumed to be of fixed sign, and thus the eigenvalues may be both positive and negative. Key ingredients in the proofs were demonstrated by Birman and Solomjak nearly fifty years ago in their seminal work on eigenvalue asymptotics. In addition to determining the eigenvalue asymptotics in the rough Riemannian manifold setting for weighted Laplace equations, we also wish to promote their achievements which may have further applications to modern problems.

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