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Cup product in bounded cohomology of the free group

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Abstract. The theory of bounded cohomology of groups has many applications. A key open problem is to compute the full bounded cohomology $\mathrm{H}_b^n(F, \mathbb{R})$ of a non-Abelian free group F with trivial real coefficients. It is known that $\mathrm{H}_b^n(F, \mathbb{R})$ is trivial for $n = 1$ and uncountable dimensional for $n = 2, 3$, but $\mathrm{H}_b^n(F, \mathbb{R})$ remains unknown for any $n \geq 4$. For $n = 4$, one may construct classes by taking the cup product $\alpha \smile \beta \in \mathrm{H}_b^4(F, \mathbb{R})$ between two 2-classes $\alpha, \beta \in \mathrm{H}_b^2(F, \mathbb{R})$. However, we show that all such cup products are trivial if α and β are classes induced by the quasimorphisms defined by Brooks or Rolli.

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