Ann. Sc. Norm. Super. Pisa Cl. Sci. (5)
Vol. XXI (2020), 133-167

Equivariant extensions of $\mathbb{G}_a$-torsors over punctured surfaces

ADRIEN DUBOULOZ, ISAC HEDÉN AND TAKASHI KISHIMOTO

Abstract. Motivated by the study of the structure of algebraic actions of the additive group on affine threefolds X, we consider a special class of such varieties whose algebraic quotient morphisms $X \to X//\mathbb{G}_a$ restrict to principal homogeneous bundles over the complement of a smooth point of the quotient. We establish basic general properties of these varieties and construct families of examples illustrating their rich geometry. In particular, we give a complete classification of a natural subclass consisting of threefolds X endowed with proper $\mathbb{G}_a$-actions, whose algebraic quotient morphisms $\pi : X \to X//\mathbb{G}_a$ are surjective with only isolated degenerate fibers, all isomorphic to the affine plane $\mathbb{A}^2$ when equipped with their reduced structures.

Mathematics Subject Classification (2010): 14R20 (primary); 14R25, 14R05, 14L30, 14D06 (secondary).