Orbital degeneracy loci and applications

VLADIMIRO BENEDETTI, SARA ANGELA FILIPPINI, LAURENT MANIVEL AND FABIO TANTURRI

Abstract. Degeneracy loci of morphisms between vector bundles have been used in a wide variety of situations. We introduce a vast generalization of this notion, based on orbit closures of algebraic groups in their linear representations. A preferred class of our orbital degeneracy loci is characterized by a certain crepancy condition on the orbit closure, that allows to get some control on the canonical sheaf. This condition is fulfilled for Richardson nilpotent orbits, and also for partially decomposable skew-symmetric three-forms in six variables. In order to illustrate the efficiency and flexibility of our methods, we construct in both situations many Calabi-Yau manifolds of dimension three and four, as well as a few Fano varieties, including some new Fano fourfolds.

Mathematics Subject Classification (2010): 14N05 (primary); 14E15, 14J32, 14J45, 14M12 (secondary).