Existence of partially localized quasiperiodic solutions of homogeneous elliptic equations on $\mathbb{R}^{N+1}$

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Abstract. We consider the equation

$$\Delta u + u_{yy} + f(u) = 0, \quad (x, y) \in \mathbb{R}^N \times \mathbb{R}, \tag{1}$$

where $N \geq 2$ and f is a smooth function satisfying $f(0) = 0$ and $f'(0) < 0$. We show that for suitable nonlinearities f of this form equation (1) possesses uncountably many positive solutions which are quasiperiodic in y, radially symmetric in x, and decaying as $|x| \to \infty$ uniformly in y. Our method is based on center manifold and KAM-type results and involves analysis of solutions of (1) in a vicinity of a y-independent solution $u^*(x)$—a ground state of the equation $\Delta u + f(u) = 0$ on $\mathbb{R}^N$.

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