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Uniqueness for unbounded solutions to stationary viscous Hamilton-Jacobi equations

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Abstract. We consider a class of stationary viscous Hamilton-Jacobi equations as

$$\begin{cases} \lambda u - \operatorname{div}(A(x)\nabla u) = H(x, \nabla u) & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega \end{cases}$$

where $\lambda \geq 0$, $A(x)$ is a bounded and uniformly elliptic matrix and $H(x, \xi)$ is convex in ξ and grows at most like $|\xi|^q + f(x)$, with $1 < q < 2$ and $f \in L^{N/q'}(\Omega)$. Under such growth conditions solutions are in general unbounded, and there is not uniqueness of usual weak solutions. We prove that uniqueness holds in the restricted class of solutions satisfying a suitable energy-type estimate, *i.e.* $(1 + |u|)^{\bar{q}-1} u \in H_0^1(\Omega)$, for a certain (optimal) exponent $\bar{q}$. This completes the recent results in [15], where the existence of at least one solution in this class has been proved.

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