Ann. Sc. Norm. Super. Pisa Cl. Sci. (5)
Vol. XX (2020), 413-414

Errata corrige. Elliptic and parabolic problems for a class of operators with discontinuous coefficients

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Abstract. The proof of Proposition 3.18 in [1] contains a mistake since an r^2 appeared erroneously, instead of r, in the equation for w. The statement is however correct and below is an amended proof.

Mathematics Subject Classification (2010): 47D07 (primary); 35B50, 35J25, 35J70 (secondary).

Propositio 3.18. *Assume that $b = c = 0$ and that $\lambda = N - 1$. Then $D_\lambda > 0$ and $s_1^{(\lambda)} = -1$, $s_2^{(\lambda)} = (N-1)/a$. If $\frac{N}{p} \in (0, s_2^{(\lambda)} + 2)$, then u'', $\frac{u'}{r} - \frac{u}{r^2} \in L^p_{\text{rad}}$ for every $u \in D(L_{\lambda,p,\text{int}})$. It follows that*

$$D(L_{\lambda,p,\text{int}}) = \left\{u \in L^p_{\text{rad}} \cap W^{2,p}_{\text{loc}}((0,\infty))\ ;\ u'', (1 \wedge r)^{-1}u', (1 \wedge r)^{-2}u \in L^p_{\text{rad}}\right\}$$

if $1 < p < N$ and

$$D(L_{\lambda,p,\text{int}}) = \left\{u \in L^p_{\text{rad}} \cap W^{2,p}_{\text{loc}}((0,\infty))\ ;\ u', u'', \frac{u'}{r} - \frac{u}{r^2} \in L^p_{\text{rad}}\right\}$$

if $N \le p < \infty$.

Proof. Note that $D_\lambda = (N - 1 + a)^2/4a^2 \ge 1$ so that no critical case occurs. Moreover $s_1^{(\lambda)} = -1$, $s_2^{(\lambda)} = (N-1)/a > 0$ are immediately verified.

If $1 < p < N$, then $s_1^{(\lambda)} + 2 = 1 < \frac{N}{p}$ and the assertions follow from [1, Proposition 3.15] since $\theta = 1$.

Let us therefore assume that $p \ge N$. For $u \in D(L_{\lambda,p,\text{int}})$, $g = L_\lambda u = au'' + \frac{N-1}{r}(u' - u/r)$ belongs to L^p_{rad}. Setting $k = \frac{N-1}{a} > 0$ we obtain $u'' + k\left(\frac{u}{r}\right)' = f$ with $f = g/a$. Let v=u/r and $w = v' = (u'/r - u/r^2)$. Then

$$w' + \frac{k+2}{r}w = \frac{f}{r}$$

and integrating between ε and r we obtain

$$r^{k+2}w(r) - \varepsilon^{k+2}w(\varepsilon) = \int_\varepsilon^r f(t)t^{k+1}\,dt.$$

The integral on the right hand side converges as $\varepsilon \to 0$, by Hölder inequality with respect to the weight $t^{N-1}dt$, and then $\varepsilon^{k+2}w(\varepsilon) \to \ell$ as $\varepsilon \to 0$. Since $u \in D(L_{\lambda,p,\mathrm{int}})$, then $w(r)r^{2-2\theta} = (u'/r - u/r^2)r^{2-2\theta} \in L^p_{\mathrm{rad}}$ with $2\theta = N/p+1-\delta < 2$ and then $\ell = 0$. Therefore

$$w(r) = r^{-k-2}\int_0^r f(t)t^{k+1}\,dt = \int_0^1 f(rs)s^{k+1}\,ds.$$

Minkowski inequality then yields

$$\begin{aligned}\|w\|_{p,\mathrm{rad}} &\le \int_0^1 s^{k+1}\,\|f(s\cdot)\|_{p,\mathrm{rad}}\,ds \le \int_0^1 s^{k+1}\,ds\left(\int_0^\infty |f(t)|^p t^{N-1}s^{-N}\,dt\right)^{\frac{1}{p}}\\ &= \|f\|_{p,\mathrm{rad}}\int_0^1 s^{k+1-\frac{N}{p}}\,ds = C\|f\|_{p,\mathrm{rad}}\end{aligned}$$

and hence $w = (u'/r - u/r^2)$ in L^p_{rad}. By difference, we obtain that $u'' \in L^p_{\mathrm{rad}}$.

To show equality (1), let us call W the space on the right hand side. We have just shown that $D(L_{\lambda,p,\mathrm{int}}) \subset W$. Since $W \subset D(L_{\lambda,p,\max})$ we have only to show that $\omega^2 - L_\lambda$ is injective on W. However this follows from [1, Equation (3.9) and Lemma 3.10] since $v_{\omega,1}$ is unbounded at infinity and $v_{\omega,2} \approx r^{-s_2^{(\lambda)}}$ as $r \to 0$, hence does not belong to L^p_{rad}, since $N/p \le 1 \le s_2^{(\lambda)}$. □

References

[1] G. METAFUNE, M. SOBAJIMA and C. SPINA *Elliptic and parabolic problems for a class of operators with discontinuous coefficients,* Ann. Sc. Norm. Super. Pisa Cl. Sci. **19** (2019), 601–654.